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ANGLES OF THE CKM UNITARITY TRIANGLE MEASURED AT BELLE

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Abstract

The Belle experiment has used several methods to measure or constrain the angles ϕ_1, ϕ_2, and ϕ_3 (or β, α, and γ) of the CKM unitarity triangle. The results are $\sin 2\phi_1 = 0.728 \pm 0.056\,(\mathrm{stat}) \pm 0.023\,(\mathrm{syst})$ or $\phi_1 = \left(23.4\,^{+2.7}_{-2.4}\right)^\circ$ from $B^0 \to J/\psi\, K^0$ decays (140 fb^{-1}); $\phi_2 = (0\text{–}19)^\circ$ or $(71\text{–}180)^\circ$ at 95.4% CL from $B^0 \to \pi^+\pi^-$ decays (253 fb^{-1}); and $\phi_3 = \left[\,68\,^{+14}_{-15}\,(\mathrm{stat}) \pm 13\,(\mathrm{syst}) \pm 11\,(\mathrm{model})\,\right]^\circ$ from $B^\pm \to (D^0, \overline{D}{}^0)K^\pm$, $(D^0, \overline{D}{}^0) \to K^0_S\,\pi^+\pi^-$ decays (253 fb^{-1}). These values satisfy the triangle relation $\phi_1 + \phi_2 + \phi_3 = 180^\circ$ within their uncertainties. The angle ϕ_1 is also determined from several $b \to s\bar{q}q$ penguin-dominated decay modes; the value obtained by taking a weighted average of the individual results differs from the $B^0 \to J/\psi\, K^0$ result by more than two standard deviations. The angle ϕ_2 is constrained by measuring a CP asymmetry in the decay time distribution; the asymmetry observed is large, and the difference in the yields of $B^0, \overline{B}{}^0 \to \pi^+\pi^-$ decays constitutes the first evidence for direct CP violation in the B system.

1 Introduction

The Standard Model predicts CP violation to occur in B^0 meson decays owing to a complex phase in the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [1]. This phase is illustrated by plotting the unitarity condition $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ as vectors in the complex plane: the phase results in a triangle of nonzero height. Various measurements in the B system are sensitive to the internal angles ϕ_1, ϕ_2, and ϕ_3 (also known as β, α, and γ, respectively); these measurements allow us to determine the angles and check whether the triangle closes. Non-closure would indicate physics beyond the Standard Model. Here we present measurements of ϕ_1 and ϕ_2 obtained by measuring time-dependent CP asymmetries, and a measurement of ϕ_3 obtained by measuring an asymmetry in the Dalitz plot distribution of three-body decays. The results presented are from the Belle experiment [2], which runs at the KEKB asymmetric-energy e^+e^- collider [3] operating at the $\Upsilon(4S)$ resonance.

In Belle, pion and kaon tracks are identified using information from time-of-flight counters, aerogel Čerenkov counters, and dE/dx information from the central tracker [4]. B decays are identified using the "beam-constrained" mass $M_{\rm bc} \equiv \sqrt{E_{\rm beam}^2 - p_B^2}$ and the energy difference $\Delta E \equiv E_B - E_{\rm beam}$, where p_B is the reconstructed B momentum, E_B is the reconstructed B energy, and $E_{\rm beam}$ is the beam energy, all evaluated in the e^+e^- center-of-mass (CM) frame. A tagging algorithm [5] is used to identify the flavor at production of the decaying B, i.e., whether it is B^0 or $\overline{B}{}^0$. This algorithm examines tracks not associated with the signal decay to identify the flavor of the non-signal B. The signal-side tracks are fit for a decay vertex, and the tag-side tracks are fit for a separate decay vertex; the distance Δz between vertices is to a very good approximation proportional to the time difference Δt between the B decays: $\Delta z \approx (\beta\gamma c)\Delta t$, where $\beta\gamma$ is the Lorentz boost of the CM system.

The dominant background is typically $e^+e^- \to q\bar{q}$ continuum events, where $q = u, d, s, c$. In the CM frame such events tend to be jet-like, whereas $B\overline{B}$ events tend to be spherical. The sphericity of an event is usually quantified via Fox-Wolfram moments [6] of the form $h_\ell = \sum_{i,j} p_i \, p_j \, P_\ell(\cos\theta_{ij})$, where i runs over all tracks on the tagging side and j runs over all tracks on either the tagging side or the signal side [7]. The function P_ℓ is the ℓth Legendre polynomial and θ_{ij} is the angle between momenta $\vec{p_i}$ and $\vec{p_j}$ in the CM frame. These moments are combined into a Fisher discriminant, and this is combined with the probability density function (PDF) for $\cos\theta_B$, where θ_B is the polar angle in the CM frame between the B direction and the z axis (nearly along the e^- beam direction). $B\overline{B}$ events are produced with a $1 - \cos^2\theta_B$ distribution while $q\bar{q}$ events are produced uniformly in $\cos\theta_B$. The PDFs for signal and $q\bar{q}$ background are obtained using MC simulation and $M_{\rm bc}$-ΔE sidebands in data,

respectively. We use the products of the PDFs to calculate a signal likelihood $\mathcal{L}_s$ and a continuum likelihood $\mathcal{L}_{q\bar{q}}$ and require that $\mathcal{L}_s/(\mathcal{L}_s+\mathcal{L}_{q\bar{q}})$ be above a threshold.

The angles ϕ_1 and ϕ_2 are determined by measuring the time dependence of decays to CP-eigenstates. This distribution is given by

$$\frac{dN}{d\Delta t} \propto e^{-\Delta t/\tau}\Big[1-q\Delta\omega+q(1-2\omega)\left[\,\mathcal{A}\cos(\Delta m\Delta t)+\mathcal{S}\sin(\Delta m\Delta t)\,\right]\Big], \quad (1)$$

where $q=+1$ (-1) corresponds to B^0 $(\overline{B}{}^0)$ tags, ω is the mistag probability, $\Delta\omega$ is a possible difference in ω between B^0 and $\overline{B}{}^0$ tags, and Δm is the B^0-$\overline{B}{}^0$ mass difference. The CP-violating coefficients $\mathcal{A}$ and $\mathcal{S}$ are functions of the parameter λ: $\mathcal{A}=(|\lambda|^2-1)/(|\lambda|^2+1)$ and $\mathcal{S}=2\,\mathrm{Im}(\lambda)/(|\lambda|^2+1)$, where

$$\lambda \;=\; \frac{q}{p}\,\frac{A(\overline{B}{}^0\to f)}{A(B^0\to f)} \;\approx\; \sqrt{\frac{M_{12}^*}{M_{12}}}\,\frac{A(\overline{B}{}^0\to f)}{A(B^0\to f)} \;=\; \left(\frac{V_{td}\,V_{tb}^*}{V_{td}^*\,V_{tb}}\right)\,\frac{A(\overline{B}{}^0\to f)}{A(B^0\to f)}\,. \quad (2)$$

In this expression, q and p are the complex coefficients relating the flavor eigenstates B^0 and $\overline{B}{}^0$ to the mass eigenstates, M_{12} is the off-diagonal element of the B^0-$\overline{B}{}^0$ mass matrix, and we assume that the off-diagonal element of the decay matrix is much smaller: $\Gamma_{12}\ll M_{12}$. If only one weak phase enters the decay amplitude $A(\overline{B}{}^0\to f)$, then $|A(\overline{B}{}^0\to f)/A(B^0\to f)|=1$ and $\lambda=\eta_f\,e^{i\,2\theta}$, where $\eta_f=\pm 1$ is the CP of the final state f. For the final states discussed here, $|\theta|=\phi_1$ or ϕ_2.

2 The angle ϕ_1

This angle is most accurately measured using $B^0\to J/\psi\,K^0$ decays[1]. The decay is dominated by a $b\to c\bar{c}s$ tree amplitude and a $b\to s\bar{c}c$ penguin amplitude. The latter can be divided into two pieces: a piece with c and t in the loop that has the same weak phase as the tree amplitude, and a piece with u and t in the loop that has a different weak phase but is suppressed by $\sin^2\theta_C$ relative to the first piece. Due to this suppression, $A(\overline{B}{}^0\to f)$ is governed by a single weak phase: $\mathrm{Arg}(V_{cb}\,V_{cs}^*)$. The ratio $A(\overline{B}{}^0\to J/\psi\,K_S^0)/A(B^0\to J/\psi\,K_S^0)$ includes an extra factor $(p/q)_K=V_{cd}^*\,V_{cs}/(V_{cd}\,V_{cs}^*)$ to account for the $\overline{K}{}^0$ oscillating to a K_S^0, and thus $\lambda=-[V_{td}\,V_{tb}^*/(V_{td}^*\,V_{tb})]\,[V_{cb}\,V_{cs}^*/(V_{cb}^*\,V_{cs})]\,[V_{cd}^*\,V_{cs}/(V_{cd}\,V_{cs}^*)]=-e^{-i\,2\phi_1}$. The CP asymmetry parameters are therefore $\mathcal{S}=\sin 2\phi_1$, $\mathcal{A}=0$. To determine ϕ_1, we fit the Δt distribution for $\mathcal{S}$; the result is $\sin 2\phi_1=0.728\,\pm 0.056$ (stat) $\pm$

[1]This measurement includes $B^0\to J/\psi\,K_S^0$, $J/\psi\,K_L^0$, $\psi(2S)K_S^0$, $\chi_{c1}K_S^0$, $\eta_c K_S^0$, and $J/\psi\,K^{*0}\,(K^{*0}\to K_S^0\pi^0)$; we use "$B^0\to J/\psi\,K^0$" to denote all six modes.

Table 1: *Decay modes used to measure* $\sin 2\phi_1$*, the number of candidate events, the value of* $\sin 2\phi_1$ *obtained, and the parameter* $\mathcal{A}$ *obtained [see Eq. (1)]. The* $B^0 \to J/\psi\, K^0$ *result corresponds to 140* fb^{-1} *of data; the other results correspond to 253* fb^{-1} *of data.*

(CP) Mode	Candidates	$\sin 2\phi_1$	$\mathcal{A}$
$(-)\, J/\psi\, K^0_S$	2285	$0.728 \pm 0.056 \pm 0.023$	–
$(+)\, J/\psi\, K^0_L$	2332		
$(-)\, \phi\, K^0_S$	139 ± 14	$0.08 \pm 0.33 \pm 0.09$	$0.08 \pm 0.22 \pm 0.09$
$(+)\, \phi\, K^0_L$	36 ± 15		
$(\pm)\, K^+K^-K^0_S$ $(+ =\ 83\%)$	398 ± 28	$0.74 \pm 0.27\,^{+0.39}_{-0.19}$	$-0.09 \pm 0.12 \pm 0.07$
$(+)\, f_0(980)\, K^0_S$	94 ± 14	$-0.47 \pm 0.41 \pm 0.08$	$-0.39 \pm 0.27 \pm 0.09$
$(+)\, K^0_S K^0_S K^0_S$	88 ± 13	$-1.26 \pm 0.68 \pm 0.20$	$0.54 \pm 0.34 \pm 0.09$
$(-)\, \eta'\, K^0_S$	512 ± 27	$0.65 \pm 0.18 \pm 0.04$	$-0.19 \pm 0.11 \pm 0.05$
$(-)\, \pi^0\, K^0_S$	247 ± 25	$0.32 \pm 0.61 \pm 0.13$	$-0.11 \pm 0.20 \pm 0.09$
$(-)\, \omega\, K^0_S$	31 ± 7	$0.76 \pm 0.65\,^{+0.13}_{-0.16}$	$0.27 \pm 0.48 \pm 0.15$

0.023 (syst), or $\phi_1 = \left(23.4\,^{+2.7}_{-2.4}\right)^\circ$ (the smaller of the two solutions for ϕ_1). The fit result for $\mathcal{A}$ yields $|\lambda| = 1.007 \pm 0.041$ (stat) ± 0.033 (syst), in agreement with the theoretical expectation. These results correspond to 140 fb^{-1} of data [8].

There are several decay modes that proceed exclusively via penguin amplitudes (e.g., $\overline{B}{}^0 \to \phi \overline{K}{}^0$ proceeding via $b \to s\bar{s}s$) or else are dominated by penguin amplitudes (e.g., $\overline{B}{}^0 \to (\eta'/\omega/\pi^0)\overline{K}{}^0$ proceeding via $b \to s\bar{d}d$) but have the same weak phase as the $b \to c\bar{c}s$ tree amplitude. This is because the penguin loop factorizes into a c, t loop with the same weak phase and a u, t loop with a different weak phase; the latter, however, is suppressed by $\sin^2\theta_C$ relative to the former and plays a negligible role. We thus expect these decays to also have $\mathcal{S} = \sin 2\phi_1$, $\mathcal{A} = 0$. There are small mode-dependent corrections ($|\Delta\mathcal{S}| \leq 0.10$) to this prediction due to final-state rescattering [9]. Table 1 lists these modes and the corresponding values [10] of $\sin 2\phi_1$ obtained from fitting the Δt distributions; Fig. 1 shows these results in graphical form. Neglecting the small rescattering corrections and simply averaging the penguin-dominated values gives $\sin 2\phi_1 = 0.40 \pm 0.13$. This value differs from the $B^0 \to J/\psi\, K^0$ world average value by 2.4 standard deviations, which may be a statistical fluctuation or may indicate new physics.

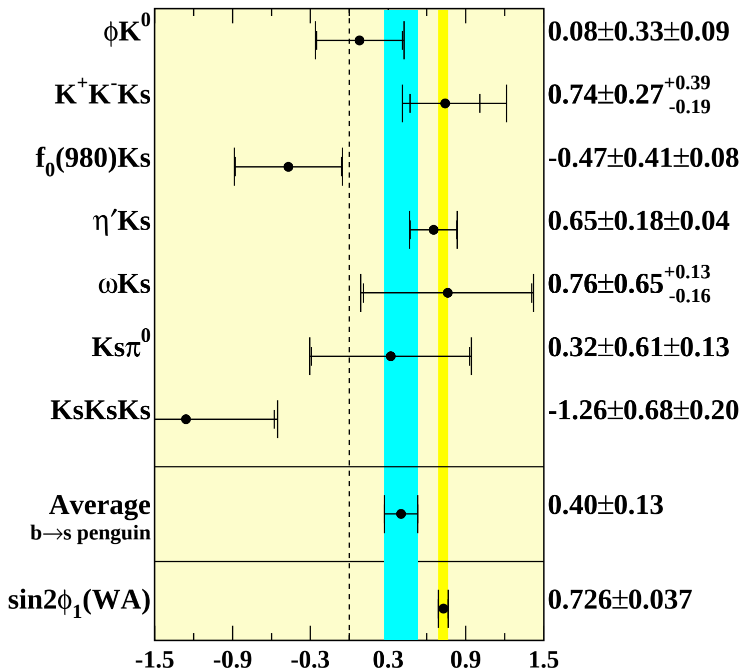

Figure 1: *Values of* $\sin 2\phi_1$ *measured in decay modes dominated by* $b \to s\bar{q}q$ *penguin amplitudes, for 253 fb*$^{-1}$ *of data. The average value differs from the world average (WA) value measured in* $B^0 \to J/\psi\, K^0$ *decays.*

3 The angle ϕ_2

This angle is measured by fitting the Δt distribution of $B^0 \to \pi^+\pi^-$ decays. The rate is dominated by a $b \to u\bar{u}d$ tree amplitude with a weak phase $\mathrm{Arg}(V_{ub}\, V^*_{ud})$. If only this phase were present, then $\lambda = [V_{td}\, V^*_{tb}/(V^*_{td}\, V_{tb})]\, [V_{ub}\, V^*_{ud}/(V^*_{ub}\, V_{ud})] = e^{i\,2\phi_2}$, and $\mathcal{S} = \sin 2\phi_2$, $\mathcal{A} = 0$. However, a $b \to d\bar{u}u$ penguin amplitude also contributes, and, unlike the penguin in $B^0 \to J/\psi K^0_S$ decays, the piece with a different weak phase is not CKM-suppressed relative to the piece with the same weak phase. The CP asymmetry parameters are therefore more complicated [11]:

$$\mathcal{A}_{\pi\pi} = -\frac{1}{R} \cdot \left(2 \left| \frac{P}{T} \right| \sin(\phi_1 + \phi_2) \sin\delta \right) \tag{3}$$

$$\mathcal{S}_{\pi\pi} = \frac{1}{R} \cdot \left(2 \left| \frac{P}{T} \right| \sin(\phi_1 - \phi_2) \cos\delta + \sin 2\phi_2 - \left| \frac{P}{T} \right|^2 \sin 2\phi_1 \right) \tag{4}$$

$$R = 1 - 2\left|\frac{P}{T}\right|\cos(\phi_1+\phi_2)\cos\delta + \left|\frac{P}{T}\right|^2 , \tag{5}$$

where $|P/T|$ is the magnitude of the penguin amplitude relative to that of the tree amplitude, δ is the strong phase difference between the two amplitudes, and ϕ_1 is known from $B^0 \to J/\psi K^0$ decays. Since Eqs. (3) and (4) have three unknown parameters, measuring $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ determines a volume in δ-$|P/T|$-ϕ_2 space.

The most recent Belle measurement [12] uses 253 fb^{-1} of data; the event sample consists of 666 ± 43 $B^0 \to \pi^+\pi^-$ candidates after background subtraction. These events are subjected to an unbinned maximum likelihood (ML) fit for Δt; the results are $\mathcal{A}_{\pi\pi} = 0.56 \pm 0.12\,(\mathrm{stat}) \pm 0.06\,(\mathrm{syst})$ and $\mathcal{S}_{\pi\pi} = -0.67 \pm 0.16\,(\mathrm{stat}) \pm 0.06\,(\mathrm{syst})$, which together indicate large CP violation. The nonzero value for $\mathcal{A}_{\pi\pi}$ indicates *direct* CP violation. Fig. 2 shows the Δt distributions for $q = \pm 1$ tagged events along with projections of the ML fit; a clear difference is seen between the fit results.

The values of $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ determine a 95.4% CL (2σ) volume in δ-$|P/T|$-ϕ_2 space. Projecting this volume onto the δ-$|P/T|$ axes gives the region shown in Fig. 3; from this region we obtain the constraints $|P/T| > 0.17$ for any value of δ, and $-180^\circ < \delta < -4^\circ$ for any value of $|P/T|$.

The dependence upon δ and $|P/T|$ can be removed by performing an isospin analysis [13] of $B \to \pi\pi$ decays. This method uses the measured branching fractions for $B \to \pi^+\pi^-$, $\pi^\pm\pi^0$, $\pi^0\pi^0$ and the CP asymmetry parameters $\mathcal{A}_{\pi^+\pi^-}$, $\mathcal{S}_{\pi^+\pi^-}$, and $\mathcal{A}_{\pi^0\pi^0}$. We scan values of ϕ_2 from $0^\circ - 180^\circ$ and for each value construct a χ^2 based on the difference between the predicted values for the six observables and the measured values. We convert this χ^2 into a confidence level (CL) by subtracting off the minimum χ^2 value and inserting the result into the cumulative distribution function for the χ^2 distribution for one degree of freedom. The resulting function $1 - \mathrm{CL}$ is plotted in Fig. 4. From this plot we read off a 95.4% CL interval $\phi_2 = (0-19)^\circ$ or $(71-180)^\circ$, i.e., we exclude the range $20^\circ - 70^\circ$.

4 The angle ϕ_3

The angle ϕ_3 is challenging to measure by fitting the Δt distribution, as the two requisite interfering amplitudes have very different magnitudes, and the small ratio of magnitudes multiplies the ϕ_3-dependent term $[\sin(2\phi_1 + \phi_3 \pm \delta)]$ [14]. As an alternative, one can probe ϕ_3 via interference in the Dalitz plot distribution of $B^\pm \to (D^0, \overline{D}{}^0)K^\pm$ decays: the additional phase ϕ_3 causes a difference between the interference pattern for B^+ decays and that for B^- decays [15].

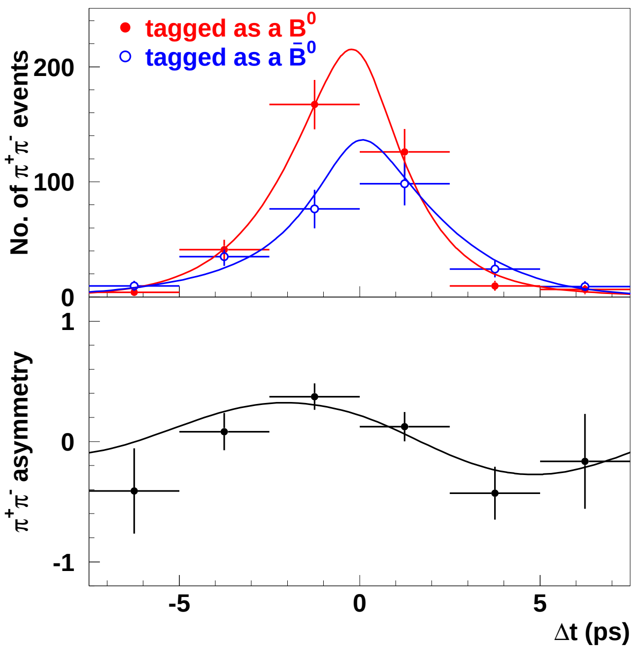

Figure 2: *The Δt distribution of background-subtracted $B^0, \overline{B}^0 \to \pi^+\pi^-$ candidates (top), and the resulting CP asymmetry $[N(\overline{B}^0) - N(B^0)]/[N(\overline{B}^0) + N(B^0)]$ (bottom). The smooth curves are projections of the unbinned ML fit.*

We study this asymmetry by reconstructing $B^\pm \to (D^0, \overline{D}^0)K^\pm$ decays in which the D^0 or $\overline{D}^0$ decays to the common final state $K_S^0\pi^+\pi^-$. Denoting $m(K_S^0, \pi^+) \equiv m_+$, $m(K_S^0, \pi^-) \equiv m_-$, $A(D^0 \to K_S^0\pi^+\pi^-) \equiv A(m^+, m^-)$, and $A(\overline{D}^0 \to K_S^0\pi^+\pi^-) \equiv \overline{A}(m^+, m^-) = A(m^-, m^+)$ (i.e., assuming CP conservation in D^0 decays), we have

$$A(B^+ \to \tilde{D}^0 K^+,\, \tilde{D}^0 \to K_S^0\pi^+\pi^-) \;=\; A(m_+^2, m_-^2) + re^{i(\delta+\phi_3)}A(m_-^2, m_+^2) \tag{6}$$

$$A(B^- \to \tilde{D}^0 K^-,\, \tilde{D}^0 \to K_S^0\pi^+\pi^-) \;=\; A(m_-^2, m_+^2) + re^{i(\delta-\phi_3)}A(m_+^2, m_1^2)\,, \tag{7}$$

where $\tilde{D}^0$ denotes $(D^0 + \overline{D}^0)$, r is the ratio of magnitudes of the two amplitudes $\left|A(B^+ \to D^0K^+)/A(B^+ \to \overline{D}^0K^+)\right|$, and δ is the strong phase difference

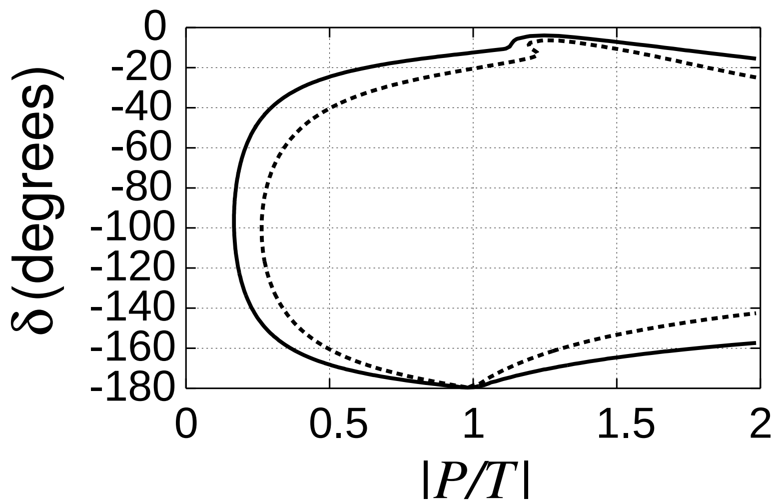

Figure 3: *Projection of the 68.3% CL (dashed) and 95.4% CL (solid) volumes in δ-$|P/T|$-ϕ_2 space onto the δ-$|P/T|$ axes. From the solid contour we obtain the constraints $|P/T|>0.17$ and $-180^\circ<\delta<-4^\circ$ (95.4% CL).*

between the amplitudes. The decay rates are given by

$$\left|A(B^+ \to \tilde{D}^0 K^+ \to (K^0_S\,\pi^+\pi^-)K^+)\right|^2 \;=\; |A(m^2_+, m^2_-)|^2 + r^2|A(m^2_-, m^2_+)|^2 + 2r|A(m^2_+, m^2_-)||A(m^2_-, m^2_+)|\cos(\delta+\phi_3+\theta) \qquad (8)$$

$$\left|A(B^- \to \tilde{D}^0 K^- \to (K^0_S\,\pi^+\pi^-)K^-)\right|^2 \;=\; r^2|A(m^2_+, m^2_-)|^2 + |A(m^2_-, m^2_+)|^2 + 2r|A(m^2_+, m^2_-)||A(m^2_-, m^2_+)|\cos(\delta-\phi_3+\theta)\,, \qquad (9)$$

where θ is the phase difference between $A(m^2_+, m^2_-)$ and $A(m^2_-, m^2_+)$ and varies over the Dalitz plot. Thus, given a $D^0 \to K^0_S\,\pi^+\pi^-$ decay model $A(m^2_+, m^2_-)$, one can fit the $B^\pm$ Dalitz plots to Eqs. (8) and (9) to determine the parameters r, δ, and ϕ_3. The decay model is determined from data, i.e., $D^0 \to K^0_S\,\pi^+\pi^-$ decays produced via $e^+e^- \to c\bar{c}$.

The data sample used consists of 253 fb^{-1}; there are 209 ± 16 $B^\pm \to \tilde{D}^0 K^\pm$ candidates with 75% purity, and an additional 58 ± 8 $B^\pm \to \tilde{D}^{0*} K^\pm$ ($\tilde{D}^{0*} \to \tilde{D}^0\pi^0$) candidates with 87% purity [16]. The background is dominated by $q\bar{q}$ continuum events in which a real D^0 is combined with a random kaon, and random combinations of tracks in continuum events. The Dalitz plots for the final samples are shown in Fig. 5.

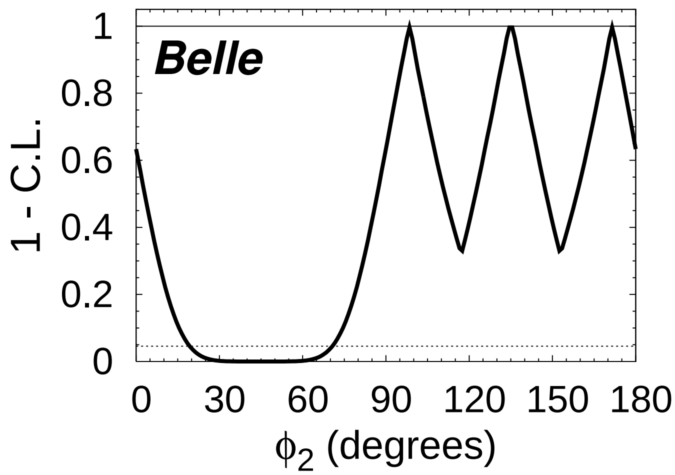

Figure 4: *The result of fitting the branching fractions for* $B \to \pi^+\pi^-$*,* $\pi^\pm\pi^0$*,* $\pi^0\pi^0$ *and the* CP *asymmetry parameters* $\mathcal{A}_{\pi^+\pi^-}$*,* $\mathcal{S}_{\pi^+\pi^-}$*, and* $\mathcal{A}_{\pi^0\pi^0}$*, as a function of* ϕ_2 *(see text). The vertical axis is one minus the confidence level. The horizontal line at* $1-\mathrm{CL} = 0.046$ *corresponds to a 95.4% CL interval for* ϕ_2*.*

The events are subjected to an unbinned ML fit for r, δ, and ϕ_3. The decay model is a coherent sum of two-body amplitudes and a constant term for the nonresonant contribution:

$$A(m_+^2, m_-^2) \;=\; \sum_{j=1}^{N} a_j\, e^{i\alpha_j} \mathcal{A}_j(m_+^2, m_-^2) \;+\; a_{\mathrm{nonres}}\, e^{i\alpha_{\mathrm{nonres}}}\,, \qquad (10)$$

where a_j, α_j, and $\mathcal{A}_j$ are the magnitude, phase, and matrix element, respectively, of resonance j; and $N = 18$ resonances are considered. The parameters a_j and α_j are determined by fitting a large sample of continuum $D^0 \to K^0_S\,\pi^+\pi^-$ decays. The dominant intermediate modes [16] as determined from the fraction $\int |a_j \mathcal{A}_j|^2\, dm_+^2\, dm_-^2 / \int |A(m_+^2, m_-^2)|^2\, dm_+^2\, dm_-^2$ are $K^*(892)^+\pi^-$ (61.2%), $K^0_S\,\rho^0$ (21.6%), nonresonant $K^0_S\,\pi^+\pi^-$ (9.7%), and $K^*_0(1430)^+\pi^-$ (7.4%).

The central values obtained by the fit are $r = 0.25$, $\delta = 157^\circ$, and $\phi_3 = 64^\circ$ for $B^+ \to \tilde{D}^0 K^+$; and $r = 0.25$, $\delta = 321^\circ$, and $\phi_3 = 75^\circ$ for $B^+ \to \tilde{D}^{*\,0} K^+$. The errors obtained by the fit correspond to Gaussian-shaped likelihood distributions, and for this analysis the distributions are non-Gaussian. We therefore use a frequentist MC method to evaluate the statistical errors. We first obtain

Table 2: *Results of the Dalitz plot analysis for* r, δ, *and* ϕ_3. *The first error listed is statistical and is obtained from a frequentist MC method (see text); the second error listed is systematic but does not include uncertainty from the* $\tilde{D}^0 \to K^0_S\,\pi^+\pi^-$ *decay model; the third error listed is due to the decay model.*

Parameter	$B^+ \to \tilde{D}^0 K^+$	$B^+ \to \tilde{D}^{*0} K^+$
r	$0.21 \pm 0.08 \pm 0.03 \pm 0.04$	$0.12\,^{+0.16}_{-0.11} \pm 0.02 \pm 0.04$
δ	$157^\circ \pm 19^\circ \pm 11^\circ \pm 21^\circ$	$321^\circ \pm 57^\circ \pm 11^\circ \pm 21^\circ$
ϕ_3	$64^\circ \pm 19^\circ \pm 13^\circ \pm 11^\circ$	$75^\circ \pm 57^\circ \pm 11^\circ \pm 11^\circ$

a PDF for the fitted parameters r, δ, ϕ_3 as a function of the true parameters $\bar{r}$, $\bar{\delta}$, $\bar{\phi}_3$. We do this by generating several hundred experiments for a given set of $\bar{r}$, $\bar{\delta}$, $\bar{\phi}_3$ values, with each experiment having the same number of events as the data, and fitting these experiments as done for the data. The resulting distributions for $\alpha_\pm = r\cos(\delta \pm \phi_3)$ and $\beta_\pm = r\sin(\delta \pm \phi_3)$ are modeled as Gaussians (G) with mean values $\bar{\alpha}_\pm$ and $\bar{\beta}_\pm$ and common standard deviation σ, and the product $G(\alpha_+ - \bar{\alpha}_+)\cdot G(\alpha_- - \bar{\alpha}_-)\cdot G(\beta_+ - \bar{\beta}_+)\cdot G(\beta_- - \bar{\beta}_-)$ is used to obtain the PDF $\mathcal{P}(r,\,\delta,\,\phi_3|\bar{r},\,\bar{\delta},\,\bar{\phi}_3)$. With this PDF we calculate the confidence level for $\{\bar{r},\,\bar{\delta},\,\bar{\phi}_3\}$ given the fit values $\{0.25,\,157^\circ,\,64^\circ\}$ for $B^+ \to \tilde{D}^0 K^+$ and $\{0.25,\,321^\circ,\,75^\circ\}$ for $B^+ \to \tilde{D}^{*0} K^+$. The resulting confidence regions for pairs of parameters are shown in Fig. 6. The plots show 20%, 74%, and 97% CL regions, which correspond to one, two, and three standard deviations, respectively, for a three-dimensional Gaussian distribution. The 20% CL regions are taken as the statistical errors; the values that maximize the PDF are taken as the central values. Of the two possible solutions (δ, ϕ_3) or $(\delta+\pi, \phi_3+\pi)$, we choose the one that satisfies $0^\circ < \phi_3 < 180^\circ$.

All results are listed in Table 2. The second error listed is systematic and results mostly from uncertainty in the background Dalitz plot density, variations in efficiency, the $m^2_{\pi\pi}$ resolution, and possible fitting bias. The third error listed results from uncertainty in the $D^0 \to K^0_S\,\pi^+\pi^-$ decay model, e.g., from the choice of form factors used for the intermediate resonances and the q^2 dependence of the resonance widths.

We combine the $B^+ \to \tilde{D}^0 K^+$ and $B^+ \to \tilde{D}^{*0} K^+$ results by multiplying together their respective PDF's, taking the parameter $\bar{\phi}_3$ to be common between them. This gives a PDF for the six measured parameters r_1, δ_1, $\phi_{3\,(1)}$, r_2, δ_2, $\phi_{3\,(2)}$ in terms of the five true parameters $\bar{r}_1$, $\bar{\delta}_1$, $\bar{r}_2$, $\bar{\delta}_2$, $\bar{\phi}_3$. The value of $\bar{\phi}_3$ that maximizes the PDF is taken as the central value, and the 3.7% CL interval prescribed by the PDF (corresponding to 1σ for a five-dimensional Gaussian distribution) is taken as the statistical error. The systematic error

is taken from the $B^+ \to \tilde{D}^0 K^+$ measurement, as this sample dominates the combined measurement. The overall result is

$$\phi_3 \quad = \quad \left[\, 68\,^{+14}_{-15}\,(\mathrm{stat})\ \pm 13\,(\mathrm{syst})\ \pm 11\,(\mathrm{decay\ model})\,\right]^{\circ}\ .$$

The 2σ confidence interval including the systematic error and decay model error is $22^{\circ} < \phi_3 < 113^{\circ}$.

In summary, the Belle experiment has measured or constrained the angles ϕ_1, ϕ_2, and ϕ_3 of the CKM unitarity triangle. We obtain $\sin 2\phi_1 = 0.728 \pm 0.056\,(\mathrm{stat}) \pm 0.023\,(\mathrm{syst})$ or $\phi_1 = \left(23.4\,^{+2.7}_{-2.4}\right)^{\circ}$ with 140 fb^{-1} of data; $\phi_2 = (0-19)^{\circ}$ or $(71-180)^{\circ}$ at 95.4% CL with 253 fb^{-1} of data; and $\phi_3 = \left[\, 68\,^{+14}_{-15}\,(\mathrm{stat})\ \pm 13\,(\mathrm{syst})\ \pm 11\,(\mathrm{decay\ model})\,\right]^{\circ}$ with 253 fb^{-1} of data. Within their uncertainties, these values satisfy the triangle relation $\phi_1 + \phi_2 + \phi_3 = 180^{\circ}$. The angle ϕ_1 is measured from $B^0 \to J/\psi\, K^0$ decays and also from several $b \to s\bar{q}q$ penguin-dominated decay modes; the value obtained from the penguin modes differs from the $B^0 \to J/\psi\, K^0$ result by $2.4\,\sigma$. The ϕ_2 constraint results from measuring the CP asymmetry coefficients $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ in $B^0 \to \pi^+\pi^-$ decays; the results are $\mathcal{A}_{\pi\pi} = 0.56 \pm 0.12\,(\mathrm{stat}) \pm 0.06\,(\mathrm{syst})$ and $\mathcal{S}_{\pi\pi} = -0.67 \pm 0.16\,(\mathrm{stat}) \pm 0.06\,(\mathrm{syst})$, which together indicate large CP violation. The nonzero value for $\mathcal{A}_{\pi\pi}$ indicates direct CP violation; the statistical significance (including systematic uncertainty) is $4.0\,\sigma$. These values also imply that the magnitude of the penguin amplitude relative to that of the tree amplitude ($|P/T|$) is greater than 0.17 at 95.4% CL, and that the strong phase difference (δ) lies in the range $(-180^{\circ}, -4^{\circ})$ at 95.4% CL. The ϕ_3 measurement is obtained from a Dalitz plot analysis of $B^{\pm} \to \tilde{D}^{(*)\,0} K^{\pm}$, $\tilde{D}^0 \to K^0_S\, \pi^+\pi^-$ decays; the statistical significance of the observed (direct) CP violation is 98%.

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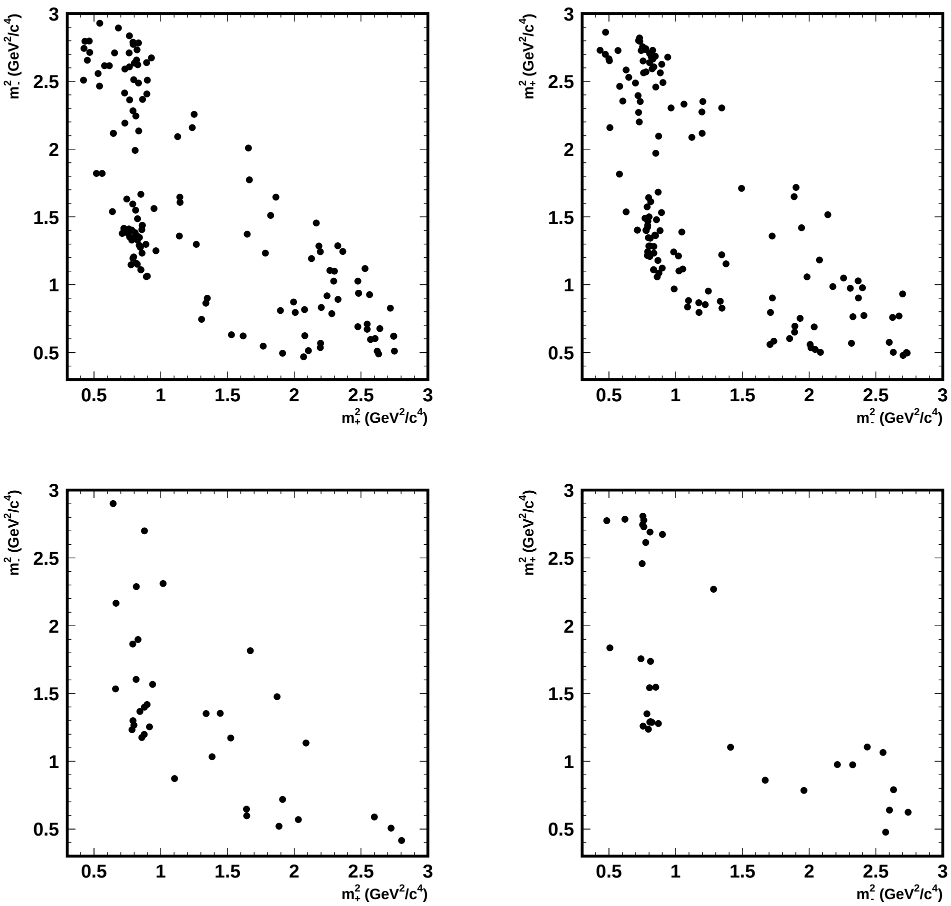

Figure 5: *Dalitz plots of $\tilde{D}^0 \to K^0_S\pi^+\pi^-$ decays obtained from samples of $B^+ \to \tilde{D}^0 K^+$ (top left), $B^- \to \tilde{D}^0 K^-$ (top right), $B^+ \to \tilde{D}^{*0} K^+$ (bottom left), and $B^- \to \tilde{D}^{*0} K^-$ (bottom right).*

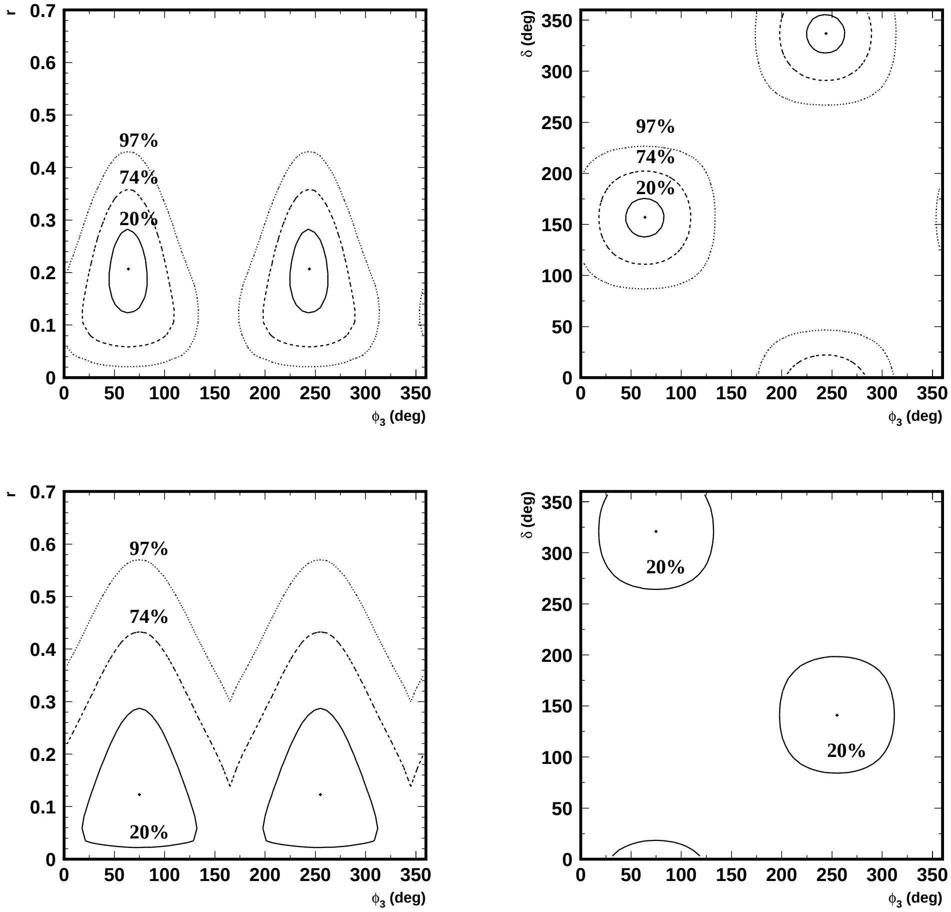

Figure 6: *Confidence regions for pairs of parameters: the left-most plots correspond to r-ϕ_3 and the right-most plots to δ-ϕ_3. The top row corresponds to $B^\pm \to \tilde{D}^0 K^\pm$ decays and the bottom row to $B^\pm \to \tilde{D}^{*0} K^\pm$ decays.*